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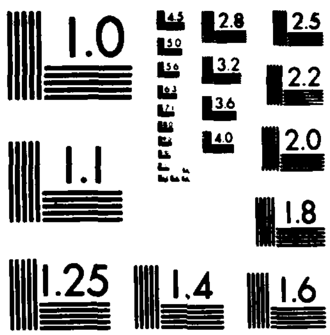

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

ORDINARY DIFFERENTIAL EQUATIONS

A three days meeting was held at University of Colorado, 20-22 May 1987. The speakers gave reports on their research to-date concerning nonlinear PDE's and possible systems of ODE's which faithfully capture their essential behavior, particularly in terms of chaotic behavior. Four of the principal speakers (ABSTRACTS) (Ercolani, McLauglin, Sell and Marsden) have AFOSR support.

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INFINITE DIMENSIONAL DYNAMICAL SYSTEMS AND THEIR FINITE DIMENSIONAL ANALOGUES

Wednesday, May 20 - Friday, May 22, 1987

Turk Seminar Room, 3rd floor Morrison

AFOSR-TR- 88-0186

Wednesday, May 20		
	9:00 a.m.	*Registration and coffee*
	9:30	*N. Ercolani* *"Sine-Gordon Phase Space - Geometry Instability"*
	10:30	*COFFEE*
	11:00	*D.W. McLaughlin* *"Coherence and Chaos in a Perturbed Sine-Gordon System"*
	12:00	*LUNCH*
	1:30	*G. Forest* *"Correlations Between the Perturbed Sine-Gordon System"*
	2:30	*COFFEE*
	3:00	*J. Poschel* *"On Infinite Dimensional KAM Theorems"*
	4:00	*Discussion period*
	5:00-7:00	*Cocktails at A.D. White House* *(No Formal Dinner Plans)*
Thursday, May 21		
	9:00 a.m.	*G. Sell* *"The Principle of Spatial Averaging and Inertial Manifolds"*
	10:00	*COFFEE*
	10:30	*C. Foias* *"Integral Manifolds: Inertial Manifolds"*
	11:30	*Discussions*
	12:00	*LUNCH*
	1:30	*B. Nicolaenko* *"Computational Aspects of Inertial Manifolds"*
	2:30	*COFFEE*
	3:00	*S. Chow* *"Inertial Manifolds"*
	4:00-4:30	*M. Brin* *"Remarks on the Ergodic Theory of Foliations and Attractors:*
	4:00-6:00	*Discussions/Additional Short Presentations* *(No Formal Dinner Plans)*
Friday, May 22		
	8:30	*S.S. Antman* *"Asympototics of Quasilinear Parabolic Equations of Viscoelasticity"*
	9:45	*J.E. Marsden* *"Exponentia[illegible] Splitting of Separatrices"*
	10:45	*COFFEE*
	11:00	*A. Mielke* *"Hamilton[illegible] and Invariant Manifolds for Elliptic Equations in Cylindrical [illegible]*
	12:00	*LUNCH*
	1:00	*N. Zabusky* *"Model Comp[illegible] the 2D Euler and Navier Stokes Equations"*
	2:30	*End of Formal [illegible]*

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INFINITE DIMENSIONAL DYNAMICAL SYSTEMS AND THEIR FINITE DIMENSIONAL ANALOGUES WORKSHOP

List of Attendees

Stuart S. Antman	University of Mathematics
Dieter Armbruster	T&AM, Cornell
I. Aronoff	CAM, Cornell
Michael Brin	Univ. of Maryland
Bjorn Birner	Univ. of Calif., Santa Barbara
Carlos Castillo-Chavez	Cornell Univ.
Dingiu Chen	Cornell University
Shui-Nee Chow	Michigan State University
Amy Novick-Cohen	Michigan State University
Joe Cusumano	T&AM, Cornell
Bo Deng	Michigan State Univ.
P.Dieft	Courant, NYU
Henk Dijkstra	Univ. of Groningen, Netherlands
Charlie Doering	Los Alamos, CNLS
Nick Ercolani	Univ. of Arizona
C. Foias	Indiana Univ.
Gilberto Flores	N.Y.U.
Greg Forest	Ohio State Univ. & C.N.L.S., Los Alamos
Paul Glendinning	Warwick/Cambridge U.K.
John Guckenheimer	Cornell Univ.
Darryl Holm	Los Alamos, MS B284
Philip Holmes	Cornell Univ.
Stewart Johnson	Cornell Univ.
Debra Lewis	Berkeley
Guangxuan Li	Cornell University
Rob Lipton	Cornell Univ.
Wei-min Liu	Cornell Univ.
Kening Lu	Michigan State Univ.
Alex Mahalov	CAM, Cornell
Jerry Marsden	Berkeley
David W. McLaughlin	Univ. of Arizona
A. Mielke	Cornell Univ.
Richard Montgomery	M.I.T.
B. Nicolaenko	MSI, Cornell Univ.
Yong-Geuh Oh	Berkeley
Peter Olver	Univ. of Minnesota
J. Poschel	Cornell Univ.
Charles Roten	Cornell Univ.
C.C.A. Sastri	Cornell Univ.
J. Scheurle	Colorado State Univ.
George Sell	Univ. of Minnesota
G.I. Mac Sithigh	Univ. Missouri-Rolla
Paul Steen	Cornell Univ.
Andrew Szeri	Cornell Univ.
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ABSTRACTS

Sine-Gordon Phase Space - Geometry and Instabilities

by

Nick Ercolani

Abstract:

The Sine-Gordon Hamiltonian system

$$\vec{u}_t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \frac{\delta H}{\delta \bar{u}}, \quad H = \int_0^L \left[\frac{u^2}{2} + \frac{v^2}{2} + 1 - \cos u \right] dx$$

on the phase space of functions $\vec{u} = \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} \in C_L^\infty \times C_L^\infty$, periodic in x of period L, is completely integrable. The complete involutive set of integrals is encoded in the coefficents of an entire function of $\lambda \in \mathbb{C}$

$$\Delta\ (\vec{u};\ \lambda)$$

which depends on $\vec{u}$. (Δ is the Floquet discriminant of a λ-eigenvalue problem whose coefficients involve $\vec{u}$.)

The modulational instabilities of Sine-Gordon are associated to the level sets of

$$\Delta:\quad C_L^\infty \times C_L^\infty \longrightarrow \{\text{entire functions}\}$$

which lie over critical values of Δ $(=\{\Delta(\lambda)\,|\,\Delta\pm 2$ has complex double roots).

Such a level set is stratified by a finite number of invariant manifolds. We show that the topology of these manifolds, how they are connected to one another, and how this topology governs the Lyapunov exponents of associated instabilities can all be determined from the function $\Delta(\lambda)$.

Spatial Coherence and Temporal Chaos in Near-Integrable PDE's

by

David W. McLaughlin

Abstract:

The damped and driven pendula chain,

$$u_{tt} - u_{xx} + \sin u = \epsilon\ [-\alpha\ u_t + \Gamma \sin \omega t],$$

is studied as a prototype model of coherence and chaos in a near integrable PDE. Both numerical and theoretical studies are summarized. The two main points are that the nearby integrable system provides (i) homoclinic orbits which act as sources of sensitivity (chaos) and (ii) candidates for approximate coordinates of the attractor. In addition, techniques founded on the nearby integrable system provide direct numerical checks on the validity and accuracy of these two points. These numerical and theoretical techniques are used to study in detail a route to chaos which involves the following characteristics:

(i) temporal - one frequency → two frequencies → chaos;

(ii) spatial - zero → one → two localized excitations;

(iii) symmetry changes and pattern competition;

(iv) low dimensional, chaotic attractors;

(v) temporal intermittancy;

(vi) homoclinic crossings;

(vii) interactions and transitions between localized and extended states

References:

1) N. Ercolani, G. Forest, D. McLaughlin (i) Lect. Appl. Math. 23, 149-165 (1986); (ii) Physica 18D, 472-474 (1986); (iii) "Homoclinic Orbits for the Periodic Sine-Gordon Equation", submitted, Physica D; (iv) "Geometry of the Modulational Instability" Parts I and II (Preprints, University of Arizona).

2) A. Bishop, M. G. Forest, D.W. McLaughlin, E. Overman, Physica D. 23, 293-328 (1986).

3) (i) A. Bishop, D.W. McLaughlin, E. Overman, to appear, Proc. of Conf. on Solitons, Tech. Univ. Denmark, Ed. by P. Christensen (1987); (ii) Physica 19D, 1-41 (1986).

4) A. Bishop, et.al. Physica 7D, 759-779 (1983).

5) N. Ercolani and M. G. Forest, Comm. Math. Phys. 99, 1-45 (1985).

6) H. McKean, Comm. Pure. Appl. Math 34, (1981).

7) Physica 7D - The entire volume; (ii) Physica 23D - The entire volume.

Correlations Between the Perturbed Sine-Gordon Equation and Finite Modal Equations

by

Greg Forest

Abstract:

In this lecture we describe: i) numerical results on the bifurcations of the damped, periodically forced Sine-Gordon equation with periodic boundary conditions, in a finely tuned parameter range; ii) an interpretation of the spatial and temperal bifurcation structures of this perturbed integrable system with regard to the exact structure of the Sine-Gordon phase space; iii) a model dynamical systems problem, which is itself a perturbed integrable Hamiltonian system, derived from the perturbed Sine-Gordon equation by a finite mode truncation in the nonlinear Schrodinger limit; and iv) the bifurcations to chaos in the four dimensional model problem.

In particular, we focus on a likely source of chaos in both the o.d.e. and p.d.e. systems: the existence of homoclinic orbits in the unperturbed integrable phase space and the continuation of these homoclinic structures in the perturbed problem. Finally we numerically correlate the homoclinic crossings in the chaotic dynamics of the full and reduced problems.

These experimental results provide physical intuition about the coexistence of simple coherent spatial structures and temporal chaos, and set the stage for carrying out the rigorous mathematical analysis to support the numerical work.

My collaborators are Nick Ercolani and Dave McLaughlin on the theoretical aspects, Alan Bishop on the formulation of the experiments and Ed Overman, Yannis Kevrekidis, Mike Jolly, Mac Hyman, and Randy Flesch on the numerical studies.

References:

1. "A quasi-periodic route to chaos in a near-integrable p.d.e.," A. Bishop, G.Forest, D. McLaughlin, and E. Overman, Physica 23D (1986), 293-328, and references therein.
2. "Correlations between the perturbed Sine-Gordon equations and finite model equations," A. Bishop, R. Flesch, G. Forest, D. McLaughlin, and E. Overman, preprint, May, 1987.
3. "Geometry of the modulational instability: local results"; "Geometry of the modulational instability: gobal results"; "Homoclinic orbits in the periodic Sine-Gordon equation", by N. Ercolani, G. Forest, and D. McLaughlin, preprints, 1987.

An Infinite Dimensional KAM - Theorem

by

J. Pöschel

Abstract:

We consider a d-dimensional lattice $\Lambda = Z^d$ of harmonic oscillators, whose frequencies ω_i, $i \in \Lambda$, are considered as parameters which may be adjusted if necessary. The unperturbed situation is described by the Hamiltorian

$$N = (\omega, y) = \sum_{i \in \Lambda} \omega_i y_i$$

in the phase space $\Pi^\Lambda \times \mathbb{R}^\Lambda$. We consider a perturbation $H = N + \epsilon P$, where P has a <u>spatial</u> <u>structure</u> (analogous to a Fourier series expansion):

$$P = \sum_{A \in \mathcal{A}} P_A ,$$

where $\mathcal{A}$ is a system of finite subsets of A, and P_A "lives on A". The size of the perturbation is expressed in terms of a <u>measure</u> μ on Λ, for example:

$$(*) \qquad \mu(A) = \sum_{i \in A} |i|$$

We then require (roughly speaking)

$$|P_A| \sim e^{-a\mu(A)},$$

with some suitable $a > 0$.

We are able to show that then, for sufficiently small ϵ, the perturbed system possesses a Cantor set of <u>infinite dimensional</u>, invariant smooth tori with linear flow on them. The frequencies ω of these tori satisfy the small division condition

$$(**) \quad |(k, \omega)| \geq \gamma\, \delta(|k|, \text{[illegible]}) , \quad 0 \neq k \in Z^\Lambda,$$

where

$$<k> = \mu(\text{supp } k)$$

$$\delta(t) = e^{-t/\log^{1+\bar{\delta}} t}, \ t \geq t_0, \ \alpha > 0.$$

We remark that for example with (*), condition (**) holds on a set of large (Gaussian) measure in ω-space $\mathbb{R}^\Lambda$.

The above generalises and improves a result by Fröhlich, Spencer and Wayne (*J. Stat. Phys.* 1986, 42).

Principle of Spatial Averaging and Inertial Manifolds

by

John Mallet-Paret and George R. Sell

Abstract:

We study the behavior of inertial manifolds for the reaction-diffusion equation

(1) $$u_t = \Delta u + f(x,u)$$

on a region $\Omega \subset R^n$ where $n \geq 2$. Our objective is to show that this equation can have an inertial manifold without the spectral gap property

$$\limsup_{n \to \infty} \lambda_{n+1} - \lambda_n = +\infty.$$

For example if $\Omega = [0,2\pi]^3$ then

$$\lambda_{n+1} - \lambda_n \leq 3.$$

for all n.

The key idea in the lecture is to introduce the Principle of Spatial Averaging and to show this principle can be used to show the existence of inertial manifolds. This theory does apply in the case where (1) is dissipative, $f \in C^3$, and $\Omega = [0,2\pi]^3$, where the boundary condition are of Dirichlet, Neumann or periodic, types.

Reference: IMA Preprint 331

Integral Manifolds: Inertial Manifolds

by

Ciprian Foias

Abstract:

Many partial differential equations describing dissipative phenomena have, when supplemented with appropriate boundary conditions, the following form

(1) $u_t + Au + R(u) = 0$

where A is a positive operator on a Hilbert space $\mathscr{H}$ with a compact inverse A^{-1} and R(u) is a differentiable operator from the domain $\mathscr{D}(A)$ of A into $\mathscr{H}$, satisfying

(2) $|R'(u)v| \leq c|Au||A^{\beta}v| \quad (u,v\in\mathscr{D}(A))$

where $c > 0$, $0 \leq \beta < 1/2$ are constants and $|\cdot|$ denotes the norm in $\mathscr{H}$. An inertial manifold of (1) is a finite dimensional (Lipschitz) manifold $\mathscr{M}$ in $\mathscr{D}(A)$, invariant to (1) (i.e. the solutions of (1) which start in $\mathscr{M}$ remain in $\mathscr{M}$) and exponentially attracting all bounded sets in $\mathscr{H}$ [1]. A natural way to construct such a manifold is to choose a large sphere Γ in the linear space generated by the eigenvectors of A corresponding to the first m distinct eigenvalues $\Lambda_1 < \Lambda_2 <\cdots< \Lambda_m$ of A, and to construct the integral manifold Σ of (1) determined by Γ; of course Γ must not be characteristic, i.e. $Au + R(u)$ should not be tangent to Γ at any $u \in \Gamma$. Finally, one shows that the closure $\mathscr{M}$ of Σ in $\mathscr{D}(A)$ is an inertial manifold. This method was developed in [2] for several remarkable partial differential equations. In [2] a central

role is played by the <u>spectral blocking property</u> of Σ. Namely, for $u \in \Sigma$ let $\Lambda(u)$ denote the maximum of (Ag,g) over all vectors g of length ≤ 1 and tangent to Σ at u. Assume that the spectral gap between Λ_m and the next eigenvalue Λ_{m+1} of A is large enough; more precisely that

$$(3) \quad \frac{\Lambda_{m+1}-\Lambda_m}{2} > c\left(\frac{\Lambda_{m+1}+\Lambda_m}{2}\right)^{\beta} \sup_{u\in\Sigma} |Au|.$$

Then if for all $u \in \Gamma$ we have

$$\Lambda(u) \leq \frac{\Lambda_{m+1}+\Lambda_m}{2},$$

this last inequality holds for all $u \in \Sigma$. A sketch of the proof of this property was given in the lecture. Some remarks and conjectures in connection with the above topics were also presented; particularly a candidate for an inertial manifold of the 2D Navier-Stokes equations was introduced.

[1] C. Foias, G.R. Sell, R. Temam: Inertial manifolds for nonlinear evolutionary equations, IMA preprint series #234 (U. of Minnesota, March 1986).

[2] P. Constantin, C. Foias, B. Nicolaenko, R. Temam: Nouveaux résultats sur les variétés inertielles pour les équations différentielles dissipatives, C.R. Acad. Sci. Paris, 302, Serie I, 1986, 375-378; Integral manifolds and inertial manifolds for dissipative partial differential equations (in preparation).

Inertial Manifolds For Dissipative Perturbed Hamiltonnian Systems

by

Basil Nicolaenko

Abstract:

We present an outline of a general theory of Inertial Manifolds for damped Hyperbolic systems and dissipatively perturbed Hamiltonian Systems. A prior lack of compactness of the relevant semi-groups raises the usual problems.

We introduce appropriately modified Hamiltonians to penorm the problem. The "strong squeezing" and "cone invariance" properties are still essential in constructing Inertial Manifolds. In contrast with previous work on parabolic dissipative systems, these "cone properties" now only make sense with respect to the metric (equivalant norm) induced by the Hamiltonian.

We give an explicit construction for a system of perturbed conservation laws modeling chaotic vapor-liquid phase changes in a compressible flow with non-convex Van-der-Waals equations of state. Computer movies show complex temporal interactions between large scale spatial structures; this suggests a reduced set of local coordinates on the corresponding inertial manifolds.

Bibliography:

1. "Integral and Inertial Manifolds", P. Constantin, C. Foias, B. Nicolaenko, R. Temam, submitted to Springer Verlag Lecture Notes in Mathematics.

2. "Inertial Manifolds for Dissipative Perturbed Hamiltonian Systems", B. Nicolaenko, (future M.S.I. Report); also submitted to "A.M.S. Contemporary Mathematics" Series.

Differentiable Foliation in Infinite Dimensional Systems

by

Shui-Nee Chow

Abstract:

We consider abstract evolution equations in a Banch space Z.

$$\dot{x} = A x + F(x) \ , \ x \in Z$$

where A generates a C^o semigroup of linear operators on Z and F is a nonlinear function. Under some general conditions, we show that if F is C^k, then there exist a C^α, $o < \alpha < 1$, foliation near $x = 0$. Furthermore, each fiber is C^k. The relation between foliations and inertial manifolds is discussed. Generalization to compact invariant sets is also considered.

References:

[1] S.N. Chow, K. Lu, and Xiao Biao Lin, Differentiable Foliations in Infinite Dimensional Systems. In preparation.

Remarks on the Ergodic Theory of Foliations and Attractors

by

M. Brin

Abstract:

The main purpose of this talk is to present some general principles of studying finite dimensional dynamical systems with attractors by means of ergodic theory and to indicate a possibility of generalzing these methods for infinite dimensional evolution equations with compact attractors. If a dynamical system has an attractor, one usually hopes that the asymptotic behavior of the system when $t \to \infty$ is very well modeled by the induced asymptotic behavior on the attractor. To make this work, one considers the set of "good" points on the attractor, i.e, those points whose orbits are correctly distributed on the attractor (in the sense of the Birkhoff ergodic theorem) and for which the linearized equation has correct asymptotic properties (in the sense of the Oseledetz multiplicative ergodic theorem). The next step is to construct the stable manifolds of "good" points and to conclude that any point in the stable manifold of a "good" point has the same asymptotic characteristics (for $t \to +\infty$) as the "good" point itself. The final step is to show that almost every (w.r.t. the Lebesgue measure) point from the basin of attraction has correct asymptotic properties. To do that one shows that the stable foliation satisfies a version of the Fubini theorem, i.e. is absolutely continuous.

The first step in generalizing this approach for the infinte dimensional case was made by D. Ruelle who constructed stable manifolds for the "good" points on a compact attractor (R. Mane proved a similar theorem for Banach spaces). Z. Nitecki and myself showed that in this case the stable foliation is absolutely continuous, which leaves some hope that the above finite dimensional approach may be generalized for some infinite dimensional evolution equations. The fact that a Hilbert space does not carry any canonical or natural measure similar to the Lebesgue measure, adds another meaning of the absolute continuity of the stable foliation since it allows one to measure the size of a set consisting of stable leaves. Such a set is big if its cross sections by finite dimensional planes transverse to the foliation have large measure. By the absolute continuity, this property does not depend on the particular cross section.

Asymptotics of Quasilinear Parabolic Equations of Viscoelasticity

by

Stuart S. Antman

Abstract:

The motion of a mass point on a spring is perhaps the most fundamental problem in the theory of oscillations. If the force exerted by the spring on the mass depends nonlinearly on the position and velocity of the mass point, then the analysis of the governing unforced differential equations is routine. A spring having these properties is effectively assumed to have zero mass. If the spring itself has mass, then its motion is coupled with that of the end mass. In the case under study the spring is assumed to be nonlinearly viscoelastic, so that its motion is governed by a third-order quasilinear "parabolic-hyperbolic" system. The coupled system is studied in the limit that the mass density of the spring goes to zero. The leading term of the resulting asymptotic expansion of the solution satisfies the system in which the spring density is zero. It is shown that the motion of the end mass typically does not satisfy an ordinary differential equation of the type described above. The formal asymptotic expansion of the solution of the full initial-boundary value problem accounting for an intial layer, can be fully justified. The justificiation relies on a number of delicate estimates for quasilinear parabolic equations.

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Exponentially Small Splitting of Separatrices

by

J. Marsden

Abstract:

Both upper and lower estimates are established for the separatrix splitting of rapidly forced systems with a homoclinic orbit. The general theory is applied to the equation

$$\ddot{\varphi} + \sin \varphi = \delta \sin \left(\frac{t}{\epsilon}\right)$$

for illustration; in this example, for any η, $0 < \eta < \pi/2$, and $0 < \epsilon \lesssim 1$, $0 \lesssim \delta \lesssim \delta_0$, for δ_0 sufficiently small, the separatrices are proved to split by an amount is no more than

$$\delta C(\eta, \delta_0) \exp \left[- \frac{1}{\epsilon} \left(\frac{\pi}{2} - \eta\right)\right]$$

where $C(\eta, \delta_0)$ is a constant depending on η and δ_0. If we replace δ by $\epsilon^p\delta$, $p \gtrsim 8$, then we have the sharper estimate

$$C_2\epsilon^p\delta e^{-\pi/2\epsilon} \lesssim \text{splitting distance} \lesssim C_1\epsilon^p\delta e^{-\pi/2\epsilon}$$

for constants C_1 and C_2. In particular, in this latter case, the Melnikov criterion correctly predicts exponentially small splitting. The techniques developed here can be applied to estimate the thickness of stochastic layers in the unfoldings of degenerate singularities and in KAM theory.

Hamiltonian Structures and Invariant Manifolds for Elliptic Equations in Cylindrical Domains

by

Alexander Mielke

Abstract:

We consider second order elliptic systems which are derived by minimizing an energy functional $I(u) = \int_\Omega W(u,\nabla u)dy$. For cylindrical domains $\Omega = (0,T) \times \Sigma$ with variables (t,x), the corresponding Euler-Lagrange equations can be written in Hamiltonian form, viz.

(*) $\dot{u} = \partial H/\partial v \ , \ \dot{v} = - \partial H/\partial u$

where $\dot{u} = \partial u/\partial t$, $v = \partial W/\partial \dot{u}$, and $H(u,v) = \int_\Sigma \{u \cdot \dot{u} - W(u,\nabla_x u,\dot{u})\}dx$ (cf.[1]).

On the other hand it is well-known that for elliptic equations on infinite cylindrical domains (i.e. $\Omega = \mathbb{R} \times \Sigma$) with Σ bounded, all small solutions lie on a finite dimensional center manifold CM (cf.[2,3]). Now it can be shown that the Hamiltonian structure of (*) carries over to a Hamiltonian structure for the ordinary differential equation describing the flow on CM. However this structure is non-canonical due to the curvature of the CM.

An Application to Saint-Venant's Problem is given [4].

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Model Complementarity for the Two-Dimensional Euler and Navier-Stokes Equations

by

Norman J. Zabusky

Abstract:

Continuum nonlinear dynamical systems in more than one space dimension are almost all mathematically intractable. For example, the generic equations of fluid dynamics, the incompressible Euler ($\nu=0$) and Navier-Stokes equations in two dimensions,

$$d_t\omega \equiv \omega_t - \Psi_y \omega_x + \Psi_x\omega_y = \nu\Delta\omega, \quad \Delta\Psi = -\omega,$$

are examples with *no* time-dependent analytical solutions if the initial condition is two or more localized vorticity distributions, $\omega(x,y,0)$ in $\mathbb{R}^{(2)}$.

A major goal of the study of nonlinear dynamical systems is an analytical understanding that will allow accurate predictions of state variables or functions of state variables over moderate-to-long times. To accomplish this objective, it is necessary to use more than one model to represent complimentary aspects of a dynamical process. The results of simulations with these models must be visualized and quantitative feature extraction must be performed using numerical diagnostic algorithms. This permits the judicious investigator to "separate" complex interactions among coherent structures into simpler and hopefully analytically tractable parts. This synergetic approach was used in the discovery of the soliton.[1,2]

The essential problems in inviscid, incompressible two-dimensional hydrodynamics were not posed until computer simulations showed: *axisymmetrization* of noncircular distributions of vorticity; *merger* of like-signed vorticity; *binding* of opposite-signed vorticity; and *entrainment* in a host vortex of small regions of irrotational fluid of opposite-signed vorticity. In all these processes one also observes *gradient intensification* of vorticity due to relative transport. Three key questions in evolving inviscid or nearly-inviscid two dimensional flows are:

(1) What are the mechanisms by which smooth vorticity distributions aggregate or "condense" into near-circular regions of vorticity (that is: the mechanisms of anixymmetrization and gradient intensification)?

(2) What are the mechanisms by which two like-signed regions of vorticity merge or the mechanisms by which two opposite-signed regions of vorticity bind or entrain?

(3) How do the filaments (small scales) which arise in axisymmetrization, merger and binding affect the long time evolution of the large-scale structures?

Axisymmetrization, merger and binding may be considered fundamental physical space interactions in two-dimensional turbulent flow.[3,4] In spectral jargon, merger corresponds to an "upward" energy cascade and filamentation accounts for the "downward" enstrophy cascade. The filamentation of vorticity during axisymmetrization, merger and binding is best understod by considering the local corotating or cotranslating streamfunction.[4]

In a recent review, Melander, Overman and Zabusky[5] discussed computational and mathematical vortex dynamics, emphasizing two-dimensional aspects. Here we focus briefly on complementary models and numerical diagnostics in this field. Beginning in the early 1970's finite-difference, spectral, pseudospectral and vortex-in-cell models and algorithms became very popular. The first three must include a dissipative mechanism to resolve small-scale structures. However, if gradient-scale lengths form which are smaller than five computational zones then significant truncation and aliasing errors arise. Thus, none of these schemes are capable of a general study of the Euler equations or even of "very high" Reynolds number flows at "moderate" times, despite the increasing availability of supercomputer resources. However, aspects of these flows can be better studied with the contour dynamical algorithms[6,7,8,9,10] and the moment model.[11] Contour dynamics, a generalization of the "waterbag" model[12] is a free-boundary-integral evolutionary method that is ideally suited for incompressible, inviscid or nearly-inviscid two-dimensional flows. The contours are the boundaries of *constant density* that are the sources of the flow; e.g., of constant vorticity regions in the homogeneous Euler equations and of constant mass density regions in the stratified Euler equations. Generally, the velocities of the contours are obtained from integrals (homogeneous Euler) or integral equations (stratified Euler, etc.) on the contours. Thus, in CD the evolution of plane curves describes the nonlocal and nonlinear dynamics of two-dimensional fluid and plasma systems. It is a natural technique for flows in unbounded media because the Green's function has a simple form. After "beyond" early times,[13, 14] one must utilize topological-change or "surgery"

algorithms which interconnect and clip contours. In effect, these introduce a smallest scale into the problem. Dritschel has automated this process with a robust algorithm which he calls "contour surgery."[14] Although it violates the Euler equation conservation laws, it seems to give very good results "up to" intermediate times since the errors are confined to the smalls + scales.

The *moment model* is derived by assuming that the vortex regions which create the flow are well-separated. A moment representation for each region is introduced and truncated after second moments (elliptical representation) and thus one obtains two additional degrees of freedom for each centroid. Although an asymptotic description, the model works well for closely interacting vorticies and resolves the initial stages of vortex merger. In particular, the model becomes integrable when applied to the symmetric merger of two identical vortex regions, and yields explicit necessary-and-sufficient conditions for merger.[15] Although the moment model is derived under the assumption of *uniform* vorticity, the conditions for merger are in agreement with our high-resolution spectral simulations containing initially *smooth* circular vortex distributions. This indicates a certain degree of universality of the merger conditions.

Recently we examined the problem of *asymmetric* merger, a more realistic problem, and thereby one richer in parameters. One may ask: which vortex "core" will be the *victor* in a merger?[16,17] It would be terribly expensive to answer the question of the vortex victor with a pseudospectral code and even with a contour dynamical code. However, the moment model gives excellent insights into the victory process and yields information for an analytical assault on the unrestricted merger problem.[18]

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